

# Research on recognition technology about the characteristics of fingerprint image basing on encode of iterated function system

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## Abstract

By research on technology about digital image basing on encode of iterated function system, IFS model of image expression is set up, and the steps and the methods about the recognition of the characteristics of fingerprint image basing on encode of iterated function system are put forward as well. With the model and the computing similar parameter between source image and result image, the characteristics of fingerprint image can be quickly acquired. Thus, the recognition of fingerprint image can be realized. The result of the example indicates that the computing velocity of IFS model is 5~10 times quicker than the traditional image matrix digital image. Meanwhile, the ratio of its recognition reaches to 99.86 percent. IFS model produces a fairly good effect by application on the identity authentication at the entrance of a network user on the platform of information network under WEB.

Algorithm IFS is based on an extension of the mathematical theory of iterated function system which permits the use of transformations which do not shrink spatial distance. It can be used on recognition technology about the characteristics of fingerprint image<sup>[1]</sup>.

## 1 IFS encode

An affine transformation  $\omega: R^2 \rightarrow R^2$  from two dimensional space  $R^2$  into itself is defined by

$$\omega \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y + b_1 \\ a_{21}x + a_{22}y + b_2 \end{bmatrix} \quad (1)$$

Where the  $a_{ij}$ 's and  $b_{ij}$ 's real constants. If  $A$  denotes the matrix  $(a_{ij})$ ,  $\vec{b}$  denotes the vector  $(b_1, b_2)'$ , where  $t$  denotes the transpose, and  $\vec{x}$  denotes the vector  $(x_1, x_2)'$ , then we write:

$$\omega(\vec{x}) = A\vec{x} + \vec{b}$$

The affine transformation is specified by six real numbers. Given an affine transformation, one can always find a nonnegative number  $s$  so that

$$\|\omega(\vec{x}) - \omega(\vec{y})\| \leq s \cdot \|\vec{x} - \vec{y}\| \quad \text{for all } \vec{x} \text{ and } \vec{y}$$

We call the smallest number  $s$  so that this is true the Lipschitz constant for  $\omega$ .

Here

$$\|\vec{x}\| = \sqrt{x_1^2 + x_2^2}$$

Such an affine transformation is call contractive if  $s < 1$ , and it is called a symmetry if

$$\|\omega(\vec{x}) - \omega(\vec{y})\| = \|\vec{x} - \vec{y}\| \quad \text{For all } \vec{x} \text{ and } \vec{y}$$

It is expansive if its Lipschitz constant is greater than one. A two-dimensional IFS consists of a set of  $N$  affine transformation,  $N$  an integer, denoted by

$$\{\omega_1, \omega_2, \omega_3, \dots, \omega_N\},$$

Each taking  $R^2$  into  $R^2$  together with a set of probabilities  $\{p_1, p_2, p_3, \dots, p_N\}$ ,

Where each  $p_i > 0$  and

$$p_1 + p_2 + p_3 + \dots + p_N = 1$$

Let  $s_n$  denote the Lipschitz constant for each  $n=1,2,\dots,N$ . Then we say that the IFS code obeys the average contractivity condition if

$$s_1^{p_1} \cdot s_2^{p_2} \cdot s_3^{p_3} \cdot \dots \cdot s_N^{p_N} < 1$$

An IFS code is an IFS

$$\{\omega_n, p_n : n=1,2,\dots,N\}$$

such that the average contractivity condition is obeyed<sup>[2]</sup>.

## 2 IFS model of image expression

Let  $\{\omega_n, p_n : n=1,2,\dots,N\}$  be an IFS code. Then by a theorem of Barnsley and Elton there is a unique associated geometrical object, denoted by  $A$ , a subset of  $R^2$ , called the attractor of the IFS. There is also a unique associated measure denoted by  $\mu$ . This measure may be thought of as a distribution of infinitely fine sand, of total mass one, lying upon  $A$ , as described intuitively above. The measure of a subset  $\beta$  of  $A$  is the weight of sand which lies upon  $\beta$ . It is denoted by  $\mu(\beta)$ . The underlying model associated with an IFS code consists of the attractor  $A$  together with the measure  $\mu$ , and is symbolized by  $(A, \mu)$ .

The structure of  $A$  is controlled by the affine maps  $\{\omega_1, \omega_2, \omega_3, \dots, \omega_N\}$  in the IFS code. That is, the  $6 \cdot N$  number in the affine maps fix the geometry of the underlying model and will in turn determine the geometry of associated images. The measure  $\mu$  is governed by the probabilities  $\{p_1, p_2, p_3, \dots, p_N\}$  in the IFS code. It is this measure which provides the rendering information for images<sup>[3]</sup>.

The underlying model  $(A, \mu)$  may be thought of as a subset of two-dimensional space whose geometry and coloration (fixed by the measure) are defined at the finest imaginable resolution. The way in which the underlying model defines images, via projection through viewing windows onto pixels, is described in the next section.

Let  $(A, \mu)$  be the underlying model associated with an IFS code. Let a viewing window be defined by

$$V = \{(x, y) : x_{\min} \leq x \leq x_{\max}, y_{\min} \leq y \leq y_{\max}\}$$

It is assumed that  $V$  has positive measure, namely  $\mu(V) > 0$ . Let a viewing resolution be specified by partitioning  $V$  into a grid of  $L \times M$  rectangles as follows. The interval  $[x_{\min}, x_{\max}]$  is divided into  $L$  subintervals  $[x_l, x_{l+1})$ , for  $l=0,1,\dots,L-1$ , where

$$x_l = x_{\min} + (x_{\max} - x_{\min}) \frac{l}{L}$$

Similarly  $[y_{\min}, y_{\max})$  is divided into  $M$  subintervals  $[y_m, y_{m+1})$  for  $m=0,1,\dots,M-1$  where

$$y_m = y_{\min} + (y_{\max} - y_{\min}) \frac{m}{M}$$

Let  $V_{l,m}$  denoted the rectangle

$$V_{l,m} = \{(x, y) : x_l \leq x < x_{l+1}, y_m \leq y < y_{m+1}\}$$

Then the digitized model associated with Vat resolution  $L \times M$  is denoted by  $\tilde{I}(V, L, M)$ . It consists of all those rectangles  $V_{l,m}$  such that  $\mu(V_{l,m}) \neq 0$ , (that is, all those rectangles upon which there resides a positive mass of sand).

The digitized model  $\tilde{I}(V, L, M)$  is rendered by assigning a single RGB index to each of its rectangles  $V_{l,m}$ . To achieve this, one specifies a color map  $f$  which associates integer color indices with real number in  $[0,1]$ . Let numcols be the number of different color which are to be used. One might choose for example nine grey tones on an RGB system; then numcols=9 and color index  $i$  is associated with  $i \cdot 12.5\%$  Red,  $i \cdot 12.5\%$  Green, and  $i \cdot 12.5\%$  Blue, for  $i=1,2,\dots,8$ . The interval  $[0,1]$  is broken up into subintervals according to

$$0=C_0 < C_1 < C_2 < \dots < C_{\text{numcols}}=1$$

Let the color map  $f$  be defined by  $f(0)=0$  and for  $x>0$  by

$$f(x) = \max\{i : x > C_i\}$$

Let  $\mu_{\max}$  denote the maximum of the measure  $\mu$  contained in one of the rectangles  $V_{l,m}$

$$\mu_{\max} = \max_{\substack{l=0,\dots,L-1 \\ m=0,\dots,M-1}} \mu(V_{l,m})$$

$\tilde{I}(V, L, M)$  is rendered by assigning color index

$$f\left(\frac{\mu(V_{l,m})}{\mu_{\max}}\right) \quad (2)$$

to the rectangle  $V_{l,m}$ .

In summary, the underlying model is converted to an image, corresponding to a viewing window  $V$  and resolution  $L \times M$ , by digitizing at resolution  $L \times M$  the part of the attractor which lies within the viewing window. The rendering values for this digitization are determined by the measure  $\mu(V_{l,m})$ , (which corresponds to the masses of sand which lie upon the pixels).

### 3 The algorithm for computing rendered images

The following algorithm starts from an IFS code  $\{\omega_n, p_n : n=1,2,\dots,N\}$  together with a specified viewing window  $V$  and resolution  $L \times M$ . It computes the associated IFS image, as defined in the previous section. In effect a random walk in  $R^2$  is generated from the IFS code, and the measures  $\mu(V_{l,m})$  for the pixels are obtained from the relative frequencies with which the different rectangles  $V_{l,m}$  are visited.

An initial point  $(x_0, y_0)$  needs to be fixed. For simplicity assume that the affine transformation  $w_1(\vec{x}) = A\vec{x} + \vec{b}$  is a contraction. Then, if we choose  $(x_0, y_0)$  as a fixed point of  $w_1$  we know a priori that  $(x_0, y_0)$  belongs to the image. This is obtained by solving the linear equation

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - A \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

An  $L \times M$  array  $I$  of integers is associated with the digitized window. A total number of iterations, num, large compared to  $L \times M$  also needs to be specified. The random walk part of the algorithm now proceeds as shown in the code of RenderIFS(). The  $L \times M$  array  $I$  is initialized to zero. After completion of the algorithm of the array  $I$  are given color index values according to Eqn.(2), i.e.

$$f\left(\frac{I[l,m]}{I_{\max}}\right)$$

Providing that num is sufficiently large, the ergodic theorem of Elton ensures that, with very high probability, the rendering value  $I_{l,m}$  assigned to the pixel  $(l,m)$  stabilizes to the unique value

defined as  $f\left(\frac{\mu(V_{l,m})}{\mu_{\max}}\right)$ . It is this algorithm which is used to

calculate all of the images given with this article. The key process of this algorithm is showed as Figure 1.

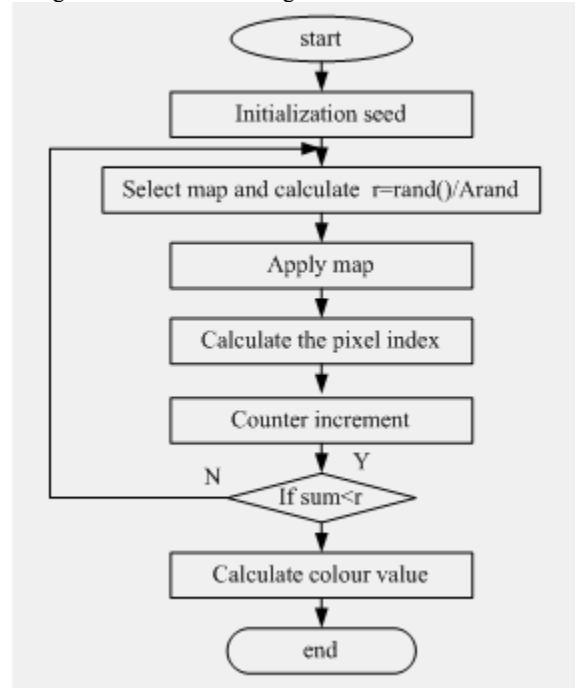


Figure 1 the key process of the RenderIFS() algorithm

### 4 The result of the experiment

In this experiment, a  $256 \times 256$  fingerprint image is used as a reference image, calculate  $64 \times 64$  reproduce is image which has a similar shape, the block is  $4 \times 4$ . The result of this experiment deduces the relationship between search step length and the peak value of SNR which is showed in the Table 1. It can be seen that the peak value of SNR increases regularly when the grads of the search step reduce<sup>[4,5]</sup>.



Source image IFS result image  
Figure.2 Source image and IFS result image

**Table 1 the relationship between search step length and the peak value of SNR**

search step length	10	8	6	4	2
search step length and the peak value of SNR	30.2252	33.9786	35.2564	37.6742	39.2236

The result via the programming example is indicated: even if under the condition that the source image is no strong with the reproduce image in comparability, we still can carry out a iteration on source image and get preferably result, expressed as the Fig.2, The Calculating rate of this model is five to ten times faster than the traditional image matrix numeric method. The rate of the image Characteristic of Recognition is 99.86%.and fairly good application of recognition rate in the figure authentication that the users log on information network platform based WEB.

## Reference:

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## Author Biography

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